FEM Analysis of Nonreciprocity of a Coplanar Waveguide with a Transversely Magnetised Ferrite Layer

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Abstract— **In this paper a study of the phase shift and characteristic impedance of a coplanar waveguide with a dielectric substrate and a ferrite superstrate is carried out using a finite element method. The results show that selecting an appropriate ratio of the ferrite layer thickness to slot width and the width of the central strip to slot width can result in larger nonreciprocity and characteristic impedance values of 40 – 60Ω.**

I. Introduction

Coplanar waveguides (CPW) or coupled slotlines loaded with transversely-magnetised ferrite layers have received considerable attention because of their nonreciprocal properties. Previous experimental and theoretical work has shown that they can be used as phase shifters, forward directional couplers, isolators and circulators[1-3]. In order to develop further this kind of component it is necessary to investigate the relationships between parameters such as the phase constant and characteristic impedance and the magnetic bias field and geometry parameters. There have been several papers on the analysis of this kind of component based on various methods. T. Kitazawa analysed propagation characteristics of coplanar-type transmission lines with ferrite and dielectric layers based on a combination of the spectral domain approach (SDA) and perturbation method, but did not calculate characteristic impedance[4]. Kitazawa and Itoh included characteristic impedance, but just for dielectric layers[5]. Slade and Webb applied the finite element method (nodal elements) to calculate the phase constants and impedance of multiconductor waveguide (including slot line and microstrips)with dielectric layers.[6]. Zhou and Davis analysed and compared properties of CPW with ferrite magnetised in the axial and transverse direction, but did not discuss impedance[7].

In this paper a study of the phase shift and characteristic impedance of a coplanar waveguide with a dielectric substrate and a ferrite superstrate (Fig.1) is carried out using a finite element method (FEM) for the first time. The results show that selecting appropriate values of the ratio of the ferrite layer thickness to slot width and the width of the central strip to slot width can result in larger nonreciprocity.

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II. Formulations

A. Formulations of the Finite Element Method (FEM)

The formulations and procedures of the FEM with edge elements for the direct calculation of phase constants for general inhomogeneously-loaded ferrite waveguides with arbitrary directions of magnetisation have been presented in [8], and therefore the details are not repeated here. It is sufficient to say that for this work an electric field (instead of magnetic field) formulation was used, and expanded to calculate the phase constants and modal impedance for the transversely-magnetised lossless ferrite CPW.

The definitions of the characteristic impedance are not unique[5,6,10,11]. Generally, three definitions may be considered (i.e. power-current type, power-voltage type and voltage-current type) and the resulting values are different. Which definition is the most suitable depends upon the structure and the application as well as the excitation and the matching conditions. For CPW with ferrite, we calculate the modal characteristic impedance only for the even mode and odd mode, and we choose the power-voltage definition. The reasons for the choice are: In most cases, only one mode (e.g. even mode , for CPW phase shifters) or , at most two modes (e.g. even and odd modes , for coupled-slot couplers or circulators) are expected to propagate; in both cases, the electromagnetic fields are concentrated in the area around the central strip and slots, and therefore calculation of the voltage between the central strip and ground planes is straightforward. Based on the above consideration, the formulation for the m-th modal characteristic impedance used in this paper is

$$Z_m = V_m^2/2P_{mz} = k_o Z_o V_m^2/2\bar{P}_{mz} \quad (1)$$

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where $Z_o = 377\ \Omega$ is the free-space wave impedance , P_{mz} is the average power propagating in the m-th mode and is given by.

$$P_{mz} = (1/2)Re\{\int_{(s)} (\mathbf{E}_m^* \times \mathbf{H}_m)_z ds\} = (1/\omega\mu_o)\bar{P}_{mz} \quad (1.1)$$

with

$$\bar{P}_{mz} = (1/2)Re\{\int_{(s)} [\mathbf{E}_m^* \times (j[\mu]^{-1} \nabla \times \mathbf{E}_m)]_z ds\}. \quad (1.2)$$

The integrals are carried out over the cross-section of the entire structure; in fact they can be carried out analytically in each element by using the expressions of the shape functions and summed over all the the elements. V_m is the maximum value of voltage of the m-th mode between the central strip and ground plane, i.e.

$$V_m = \int_{(slot)} \mathbf{E}_m \cdot d\ell \quad (1.3)$$

the integral is carried out along the shortest path between the central strip and ground plane.

III. Numerical Results and Discussion

A. Comparison with the Previous Work

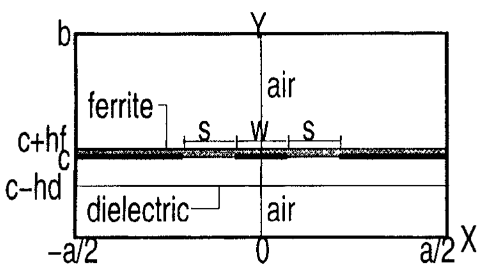

Fig 1 Cross-section of a coplanar waveguide with dielectric substrate and ferrite lay layer

Coplanar waveguide structures loaded with a dielectric substrate and a ferrite superstrate shown in Fig.1 have been simulated and analysed. In the calculation refined meshes with 150-180 nodal variables and 450-540 edge variables are used. The parameters, a, b, c, w, s, h_d and h_f are changeable and in Figs.4-6, $a = c = h_d = b/2 = 2mm$. The dielectric material is GaAs ($\varepsilon_d = 10.8$) and the ferrite is YIG ($\varepsilon_f = 13.0, 4\pi M_s = 1780\ G$), TT2-3500 ($\varepsilon_f = 12.8, 4\pi M_s = 3500G$) or TT2-111 ($\varepsilon_f = 12.5, 4\pi M_s = 5000G$)[12]. A measure of the nonreciprocity of the CPW is the differential phase shift, defined by $\phi = \beta_+ - \beta_-$ (for the same H_o) , which is the difference in phase shift per unit length for a wave travelling in the positive and negative z directions. Previous work[7,8] has proved the accuracy

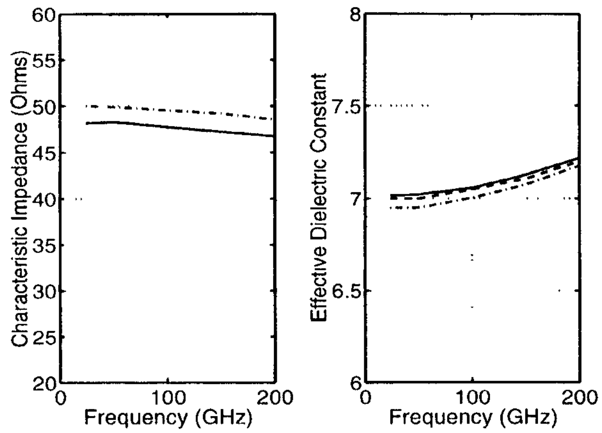

Fig. 2. Comparison of calculated (solid lines) and previous-published theoretical results (dot-dashed lines for FEM and dashed line for SDA): a) Characteristic impedance vs. frequency ; b) Effective dielectric constant vs. frequency Parameters are: $a = 0.1\ mm$, $b = 0.0402\ mm$, $c = h_d = 0.0202\ mm$, $s = 0.02\ mm$, $w/s = 0.5, h_f = 0, \varepsilon_d = 13.0$.

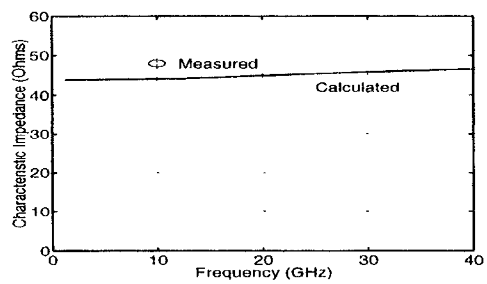

Fig. 3. Comparison of calculated and measured results. Parameters are: $a = c = b/2 = 2\ mm$, $s = 0.125\ mm$, $w = 0.3\ mm$, $h_d = 0.25\ mm$, $h_f = 0$, $\varepsilon_d = 10.8$

and effectiveness of this FEM for the calculation of propagation constants. In order to validate the calculation of impedance, the CPW structures shown in Fig.1, but without the ferrite ($h_f = 0$) have been analysed and the results are shown in Figs.2 and 3. Fig.2(a,b) present comparison of our results (solid lines) with previously-published theoretical values[11] for an MMIC structure, using the same impedance definition, up to 200 GHz Fig.3 shows a measured value[13] and values calculated by us for a larger structure. Good agreement between our results and other theoretical and measured values is evident. The small differences between may be due to the assumption of zero conductor thickness in these calculations.

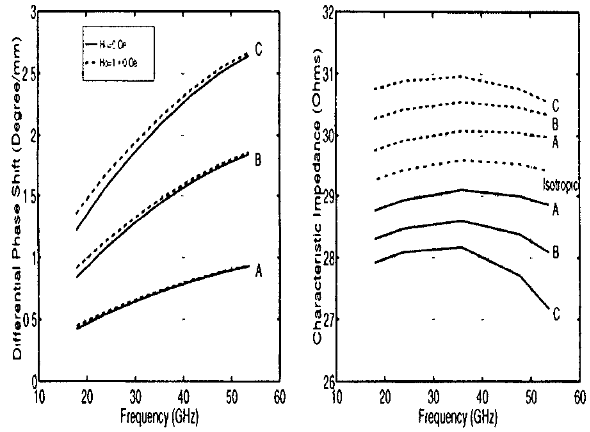

Fig 4. Effects of magnetic parameters on nonreciprocity. $s = 0.125\ mm, w/s = 2.4, h_f/s = 0.875$, for these values of $4\pi M_s$: A-1780 G; B-3500 G; C-5000 G.
(a) Differential phase shift per unit length against frequency
(b) Characteristic impedance against frequency. Forward wave —, Reverse wave - - -

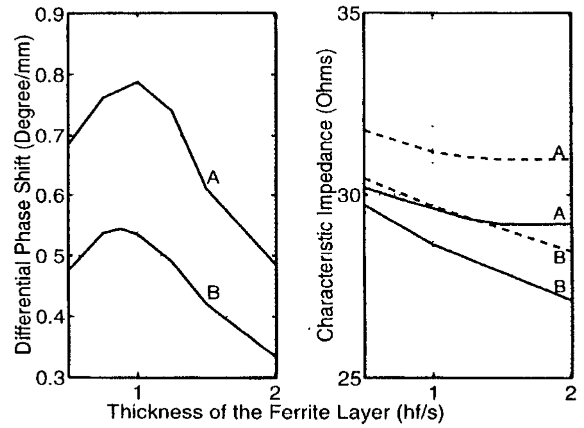

Fig. 5. Effects of the thickness of the ferrite layer on nonreciprocity. $Frequency = 24\ GHz$ and $4\pi M_s = 1780\ Oe, H_o = 0, w/s = 2.4$, A: $s = 0.18\ mm$, B: $s = 0.125\ mm$
(a) Differential phase shift of the even mode against h_f/s;
(b) Characteristic impedance of the even mode against h_f/s Forward wave —, Reverse wave - - -

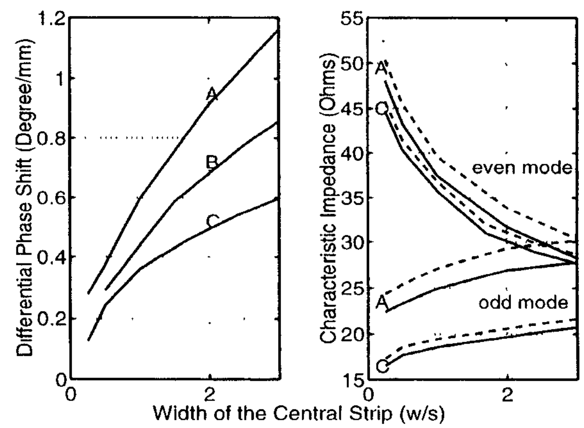

Fig 6. Effects of width of the central strip on nonreciprocity. $Frequency = 24GHz$, $4\pi M_s = 1780\ Oe, H_o = 0,\ h_f/s = 1$. A. $s = 0.24\ mm$, B: $s = 0.18\ mm$, C: $s = 0.125\ mm$)
(a) Differential phase shift of the even mode against (w/s) ,
(b) Characteristic impedance of the even mode and odd mode against (w/s) Forward wave —, Reverse wave - - -.

B. Analysis of Nonreciprocity of CPW with a Transversely Magnetised Ferrite

Figs.4(a,b) illustrate differential phase shift and characteristic impedance against frequency with three materials ($4\pi M_s$ = 1780, 3500 and 5000 G) and two bias magnetic field ($H_o = 0$ and 1000 Oe). It can be seen that:

1) Differential phase shift per unit length increases when the frequency is increased for the same material and same bias field.

2) For the same material the differential shift increases slightly when the bias field is increased from 0 Oe (just saturated) to 1000 Oe, and the higher the saturation magnetisation, the larger the effect of increasing H_o.

3) For the same bias , the material of higher saturation magnetisation provides higher nonreciprocity; for example when $4\pi M_s$ is increased from 1780 G to 5000 G, the differential phase shift is approximately tripled.

4) The characteristic impedance also exhibits nonreciprocity, because the forward and reverse waves have different field and power distributions. For the same frequency , and with $4\pi M_s = 1780\ G$, the difference of impedance for the forward and reverse waves is approximately 1 Ω. This difference increases to 3 Ω with $4\pi M_s = 5000\ G$. Thus , the variation of impedance from reciprocal to nonreciprocal is small, which means that the impedance of CPW structures is mainly decided by the geometry parameters (s, w, h_f and h_d) and dielectric parameters (ε_d and ε_f) , not by the magnetic parameters ($4\pi M_s$ and H_o). Figs.5(a,b) show the variation of differential phase shift and characteristic impedance of the even (CPW) mode with the thickness of the ferrite layer, h_f/s. It can be seen from Fig.5a that the phase shift reaches a maximum when thickness of the ferrite layer, h_f is approximately equal to the width of the slot, s. Fig.5b shows that the characteristic impedance of the CPW is decreased slightly if the thickness of the ferrite layer is increased. Figs.6(a,b) present differential phase shift and characteristic impedance against the width of the central strip relative to the width of the slot It can be seen that the differential phase shift per unit length increases as the ratio w/s is increased, and the impedance values of

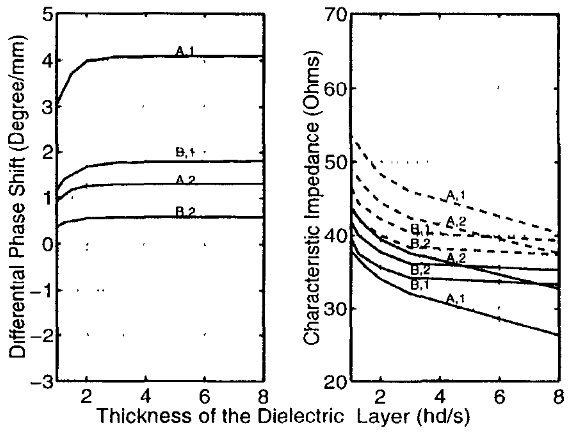

Fig. 7. Effects of thickness of the dielectric substrate on nonreciprocity. *Frequency* $= 24GHz$, $H_o = 0$ and A: $s = h_f = w/2 = 0.5\ mm$ and $a = c = b/2 = 4mm$, B: $s = h_f = w = 0.24\ mm$ and $a = c = b/2 = 2mm$, 1: $4\pi M_s = 5000\ Oe$, 2: $4\pi M_s = 1780\ Oe$.
(a) Differential phase shift of the even mode against (h_d/s) ,
(b) Characteristic impedance of the even mode against (h_d/s).
Forward wave —, Reverse wave - - -

the even mode and odd mode approach each other. Therefore, it is clear that when choosing the ratio w/s , structural compromises are necessary in order to maximise the differential phase shift while maintaining an appropriate impedance value.

Figs.7(a,b) present differential phase shift and characteristic impedance against the thickness of the dielectric substrate layer (h_d/s) It can be seen that the differential phase shift and characteristic impedance vary sharply with h_d/s less than about 2 and vary only slightly with h_d/s larger than about 3. It is noteworthy that the phase shift approaches a larger, stable value when h_d/s is increased above 2; while impedance approaches a value close to 45 Ω when h_d/s is decreased below 2 with $4\pi M_s = 1780\ G$ for this specific structure. Being similar to Fig.4, a larger saturation magnetisation causes a bigger differential phase shift and a larger variation of impedance from the isotropic value. Similar to Figs.5-6, a larger slot width may provide a bigger differential phase shift.

In summary, it can be seen from Figs.5-7 that a larger differential phase shift is obtained with a set of geometry parameters consisting of a larger w/s, h_d/s, s and an appropriate h_f/s $(= 1)$. But a reasonable value of impedance ($\sim$ 50 Ω) is obtained with smaller values of w/s and h_d/s. Therefore in order to maximise the differential phase shift and maintain an impedance value close to 50 Ω) compromises are required.

Based on the above analysis a differential phase shifter operating at $f = 24\ GHz$ with parameters $s = hd = hf = w/2 = 0.5\ mm$, $\varepsilon_d = 10.8$, $\varepsilon_d = 12.5$, $4\pi M_s = 5000\ G$, and $H_o = 0$ can provide 4 $Degree/mm$ with a characteristic impedance of $40 - 47\ \Omega$.

IV. Conclusions

In this paper a study of the phase shift and characteristic impedance of a coplanar waveguide with a dielectric substrate and a ferrite superstrate has been carried out using a finite element method. The results show that an appropriate ratio choice of ratio of the ferrite layer thickness to slot width and the width of the central strip to slot width can result in larger nonreciprocity and reasonable impedance.

V. ACKNOWLEDGMENTS

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